Recent Developments on Time-Dependent Calculation of Nonequilibrium Flows

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Nomenclature

C = mass fraction
C_v = specific heat at constant volume
e = specific internal energy
H_T = specific total enthalpy
i = ith species
L = total number of species
p = pressure
R_N = nose radius
t = time
T = temperature
$\mathbf{U}$ = velocity
δ = shock stand-off distance
ε = specific total internal energy
ω = chemical rate of production
ρ = density
θ = angular coordinate
∞ = freestream condition

Introduction

NUMERICAL studies of flow problems around a body or inside a nozzle, with or without complex chemistry require a proper formulation prior to utilization of numerical techniques and verification of the results. The problem formulation usually includes the choices of dependent and independent variables, of an appropriate coordinate system, and of an appropriate form of governing equations. In the case of time-dependent, finite-difference calculations, which are based on the fundamental physical laws, this consideration is critical to the success of the computational effort because a poor formulation will give unsatisfactory solutions and often leads to inconclusive and confusing results. In the past, much attention has been directed toward this area. Lapidus[1] indicated in the study of blunt body flows that the conservative form (or divergent-free form) of the flow equations provides more accurate solutions than a nonconservative form. Moretti,[2] however, suggested a different set of governing equations for the same flow problem, provided the shock is treated as a boundary of the computational region. He contended that the nonconservative form is preferable if the shock is determined by a fitting procedure. No attempt will be made to resolve the dispute. Instead, we intend to discuss some pecularities that are associated with the chemical nonequilibrium flows past a blunt body at high angles of attack.

The need to re-examine the basic formulation used in time-dependent, nonequilibrium flow analyses originated from the failure of an analysis described in Ref. 3 to provide satisfactory solutions in the nose region at hypersonic speeds. It was noted that some of the flow variables obtained did not conserve total enthalpy, and furthermore, the steady solution exhibited a slight dependence on initial conditions. These difficulties have prevented an extensive application to generate starting solutions for nonequilibrium, afterbody flowfield computations by means of a method of characteristics technique developed by Rakich and Park.[4] After considerable numerical experimentation, the cause of the difficulties was attributed to the use of temperature equation, being an ill-suited form for time-dependent finite-difference techniques. The present formulation to be discussed below represents a minor modification to the previous analysis; however, it results in much more accurate and reasonable solutions.

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Time-Dependent Analysis

Various forms of the energy equation can be employed in the calculation of nonequilibrium flows, but in practice the temperature equation is generally chosen for convenience. For instance, in the nozzle flow analysis by Anderson[5] and in the blunt body flow analysis by Li,[3] the temperature equation is given by

$$\frac{dT}{dt} + \frac{1}{C_v}\left(\frac{p}{\rho}\operatorname{div}\mathbf{U} + \sum_{i=1}^{L}\omega_i e_i\right) = 0 \tag{1}$$

this equation and the rate equations

$$dC_i/dt = \omega_i(T, C_i) \tag{2}$$

and the continuity, and the momentum equations are solved by a time-dependent technique[3] in which solutions are marched forward in time until the steady solution is reached. The steadiness of the solution is indicated by a negligible difference between solutions at two consecutive time steps, or simply by the magnitude of shock speed, that continuously decreases with the increasing number of time steps and by comparing the chemical composition at the wall with equilibrium values.

There were two major difficulties that hinder the usefulness of this formulation for nonequilibrium flow analysis at high speed, especially in the hypersonic range. First, it was observed that the relaxation to a steady state from prescribed initial conditions was very slow (the effect of initial flow conditions seemed to linger for a large number of time steps). Second, as a check, the total enthalpy was computed at each mesh point using T and C_i, and up to 20% error was found. This may be explained by the fact that additional equations such as the Dalton equation and the conservative equation of atomic nuclei are used to determine the species concentrations. Consequently, the third term in Eq. (1) does not represent the chemistry effect correctly.

All these difficulties disappeared when a different form of energy equation was used. It was given by

$$\frac{d\varepsilon}{dt} + \frac{1}{\rho}\operatorname{div}(p\mathbf{U}) = 0 \tag{3}$$

Where $\varepsilon = e + \frac{1}{2}\mathbf{U}\cdot\mathbf{U}$. Equation (3) is not in a conservative form, but is much closer than the form of Eq. (1). T is calculated after knowing ε, $\mathbf{U}$ and C_i at the end of each time step. Since e is expressed in terms of T, the Newton iteration procedure is used to obtain T. In so doing, the total internal energy, ε, is conserved at all mesh points. Experience with the new formulation [Eq. (1) being replaced by Eq. (3) in Ref. 3] is very satisfactory. The rate of convergence is rapid and independent of the initial conditions, the error in the computed total enthalpy decreases sharply as the solution approaches steady state, and an equilibrium flow is obtained along the body.

Shock Fitting

The essence of fitting a bow shock is to match the flow variables given by the Rankine-Hugoniot (RH) relations with the solution obtained from the time-dependent analysis. In Ref. 3 the time-dependent solution is obtained with a finite-difference (FD) scheme in which the space derivatives normal to the shock are approximated by one-side difference formulas. Shock speeds are then introduced to the RH relations so that the FD and the RH solutions agree for a selected flow variable. The component of velocity normal to the shock, the temperature or the total internal energy, and the pressure, are equally successful in the shock-fitting procedure for an ideal gas flow. The density, however, is not suitable for the matching because it has a maximum value for a given ratio of specific heats, whereas no ceiling can be imposed to the FD solution of density. It is also found that in the computation of blunt body flow at large incidence using a coordinate system in which all meridional planes emanate from the axis of the body, the normal component of velocity does not function as well as the pressure in the fitting procedure. More restriction is noted for nonequilibrium flow calculations due to the fact that the temperature or internal energy computed by the FD scheme at the shock contains the chemistry effect, which is neglected in the RH relations. If temperature is used to fit the shock, shock speed and location cannot be determined correctly and the solution finally becomes unstable. Our experience indicated that the pressure is the variable least affected by the chemistry and by the shock slope for both ideal and nonequilibrium flows at high angles of attack.

Discussion of Results

Equation (3) has been implemented in a 3D blunt body program and used to compute the flowfield around a sphere in a freestream of $V_\infty = 22{,}000$ fps, $P_\infty = 0.2118$ lb/ft^2 and $T_\infty = 426.7$°R. The nonequilibrium chemistry of air is modeled by seven species including NO^+ and e, and by 19 reactions. The thermodynamic properties of species are given in polynomials of T. The results obtained from the previous and the present formulations were compared using the same mesh size and number of time steps. However, the initial conditions were chosen differently since the previous formulation requires more realistic flow variables especially at the body in order to enhance the accuracy of the steady solution, hence, an equilibrium temperature and composition was assumed there. The present formulation is more flexible and can be started from less realistic initial conditions.

Both of the formulations use the same amount of computing time and the flowfield solutions are identical. An extra 20% of time is needed for a nonequilibrium flow calculation due to the iterations for temperature in the present formulation.

Figure 1 shows that the shock stand-off distances obtained are markedly different. The shock is closer to the body when calculated by the previous formulation, since the flow variables at the wall are defined initially to be equilibrium. Another calculation was made with the initial conditions at the wall given by an ideal gas flow. The shock stand-off distance was greater after the same number of time steps. In view of the reluctance of the solution to approach an equilibrium state at the wall and the relatively larger degree of inconsistencies among flow variables which results in a higher percent of error in the total enthalpy, the previous formulation is considered to be inaccurate. However, the shock slopes are quite close between two solutions, as are the temperatures behind the shock. Similar temperature profiles are also found along the line at $\theta = 45°$, next to the outflow boundary, and shown in Fig. 2. It has a frozen value at the shock and reduces to the equilibrium value at the wall. Nevertheless, the computed total enthalpy deviates up to 10% from its freestream value. The error in the total enthalpy is considerably less in the solution obtained from the present formulation, and continuously decreases with more time steps. H_T is conserved at the shock because the same shock fitting procedure is used for both calculations.

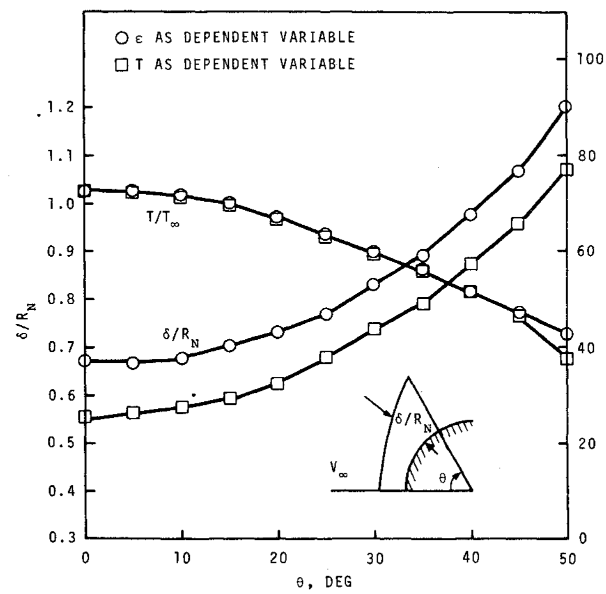

Fig. 1 Comparison of shock-standoff distance and temperature behind the shock.

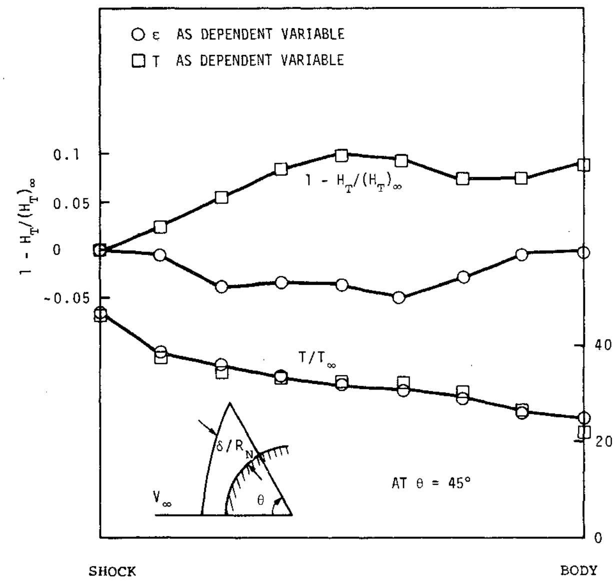

Fig. 2 Comparison of temperature and total enthalpy at a mesh line next to the outflow boundary.

The first case reported in Ref. 3 was again calculated using the modified program for freestream conditions of $V_\infty = 11{,}310$ fps, $p_\infty = 21.17$ lb/ft^2, and $T_\infty = 540$°R. Differences in shock locations and the over-all accuracy of the solutions between the present and previous formulations are almost negligible. Thus, the selection of the form of energy equation is not as critical for studying supersonic flows with low level of dissociation. This also leads to the conjecture that inconsistencies among flow variables may also exist in the time-dependent analysis of nozzle flows, especially in the region upstream of the throat.

This Note shows that erroneous and unstable solutions could result due to utilization of temperature equation and of flow variables other than pressure in fitting the bow shock in a time-dependent, nonequilibrium flow analysis. The present formulation has been critically evaluated by comparison with a previous one and found to yield steady and more accurate solutions in conserving total enthalpy. A complete assessment of the validity of the present analysis has not been made due to the lack of published experimental or numerical results, yet the discussion presented herein is probably of interest from the computational standpoint, and will shed light on future applications of time-dependent techniques.

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Analysis of Thickness Tapered Booms for Space Applications

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Nomenclature

A_x, A_o = Transverse accelerations as experienced at x and the tip, in./sec^2
$f(x)$ = Function of x corresponding to thickness profile
S_{1x}, S_{2x} = Bending stresses for constant and linearly varying accelerations (expressed as % values of uniform boom of thickness t_1)
S_1, S_2 = Maximum values of S_{1x} and S_{2x}
t_o, t_1, t_x = Wall thickness at tip, root and x, in.
T = Thickness parameter = $(t_1/t_o) - 1$
W_R = Mass ratio, i.e., boom mass expressed as percentage of the mass of a boom of uniform thickness t_1
x = Dimensionless distance (= distance from the tip divided by the boom length)
δ_1, δ_2 = Tip deflections (expressed as percentage of the values of a uniform boom of thickness t_1) corresponding to uniform and linearly varying accelerations

Introduction

SPACE booms are usually in the form of thin-walled open section tubes called STEMS[1] (Storable Tubular Extendible Members). Such a boom is stored on a cassette during critical periods of the flight such as launch and deployed in orbit. In the fully deployed position the loads experienced by a boom are mainly inertial loads due to orbital maneuvers. In general the boom experiences constant and/or linearly varying transverse accelerations along the length. Centrifugal action of a spinning spacecraft also causes transverse loads if the boom is not mounted perpendicular to the spin axis. In all these cases the loading at any point will be directly proportional to the mass outboard to that point. Since the boom under such loads acts like a cantilever beam, it suggests that providing a taper on the wall thickness can be an effective way of achieving both a low weight and an improved functional performance of the boom. The description including design and flight experience of tubular booms used for various space applications is given in Ref. 2.

The purpose of this investigation is to analyze the advantages of thickness tapered booms in terms of reductions in weight, stress and the tip deflection, especially when they are used as antennas.‡ Deflections should be minimized for improved performance of antennas. Examples of open section booms and schematic diagram of a thickness tapered boom are given in Fig. 1. Thickness variation can be obtained by a chemical etching process or by nesting several boom elements to different lengths. Two practically feasible profiles corresponding to linear and parabolic variation of the thickness along the length have been analyzed. This Note is intended for design purposes.

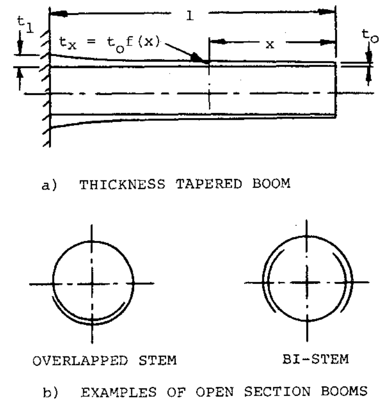

Fig. 1 A thickness tapered boom and examples of open section booms.

Analysis

For thin walled booms, the mean diameter can be assumed constant along the length as the wall thickness at any section is very small compared to the diameter. Considering the thickness profile as shown in Fig. 1a, the bending stiffness at any section x is directly proportional to t_x. The boom mass can be expressed as follows:

$$W_R = (100 t_o/t_1) \int_o^1 f(x)\,dx \qquad (1)$$

For minimum weight objective and practical consideration $t_x' \geq 0$ for $0 \leq x \leq 1$ ($'$ denotes differentiation with respect to x).

If the loading is due to transverse acceleration A_o constant through the length, using the simple beam bending theory, tip deflection and stress developed at any section can be expressed as follows:

$$\delta_1 = 800 \int_o^1 \left[G(x) - \int_o^x G(x)\cdot dx\right]\cdot dx \qquad (2)$$

$$S_{1x} = 200 \cdot G(x) \qquad (3)$$

where

$$G(x) = \left[\int_o^x \int_o^x f(x)\cdot dx \cdot dx\right] \Big/ f(x)$$

If the loading is caused by a linearly varying acceleration expressed as $A_x = A_o(1-x)$, the following relations can be obtained for bending stress and tip deflection

$$S_{2x} = 300 \cdot G_1(x) \qquad (4)$$

$$\delta_2 = (1200/11) \int_o^1 \left[G_1(x) - \int_o^x G_1(x)\cdot dx\right]\cdot dx \qquad (5)$$

where

$$G_1(x) = G(x) - \left[\int_o^x \int_o^x x\cdot f(x)\cdot dx\cdot dx\right] \Big/ f(x)$$

Equations 1–5 can be used to determine the effectiveness of any thickness tapered boom when subjected to inertial loading. Linear and parabolic thickness profiles have been evaluated using these relations.

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‡ Like the Gemini recovery antenna and Apollo 17 high frequency sounder antennas, etc.